Lecture 3
Drained and Undrained Analysis and Consolidation Considerations

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Some of the used material was originally created by:
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Contents

Drained / undrained

- Drained analysis appropriate when
  - Permeability is high
  - Rate of loading is low
  - Short term behaviour is not of interest for problem considered

- Undrained analysis appropriate when
  - Permeability is low and rate of loading is high
  - Short term behaviour has to be assessed

Drained / undrained

Suggestion by Vermeer & Meier (1998) for deep excavations:

$T < 0.10$ ($U < 10\%$) use undrained conditions

$T > 0.40$ ($U > 70\%$) use drained conditions

$$\mathbf{T = \frac{k\,E_{oed}}{\gamma_w\,D^2}\,t}$$

k = Permeability
E_{oed} = Oedometer modulus
γ_w = Unit weight of water
D = Drainage length
t = Construction time
T = Dimensionless time factor
U = Degree of consolidation

Undrained behaviour

Implications of undrained soil behaviour:

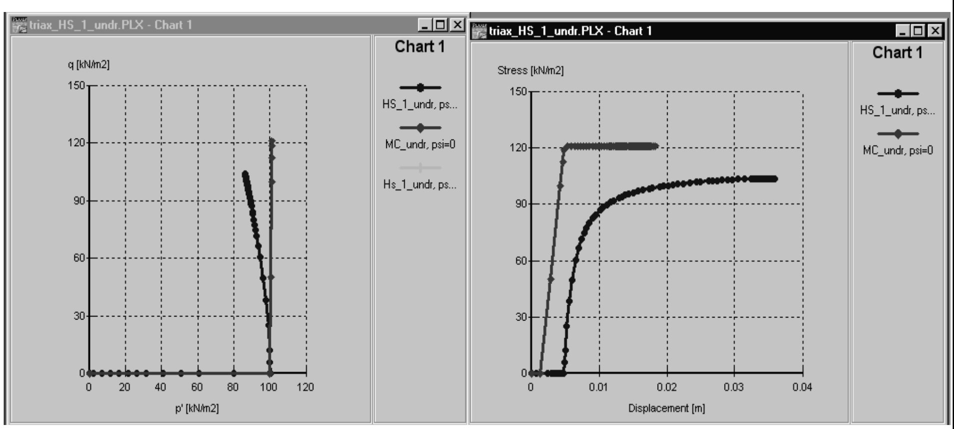
- Excess pore pressures are generated
- No volume change

    In fact small volumetric strains develop because a finite (but high) bulk modulus of water is introduced in the finite element formulation

- Predicted undrained shear strength depends on soil model used
- Assumption of dilatancy angle has serious effects on results

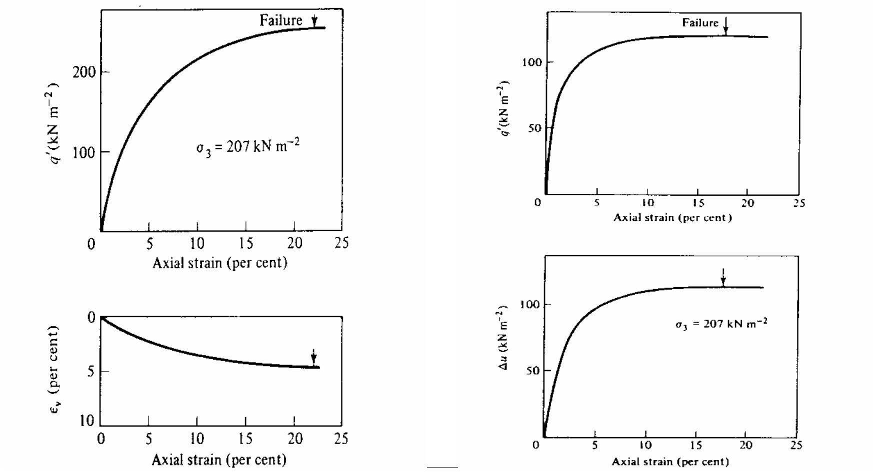
Undrained behaviour

Results from undrained triaxial tests using the Mohr-Coulomb and Hardening Soil Model



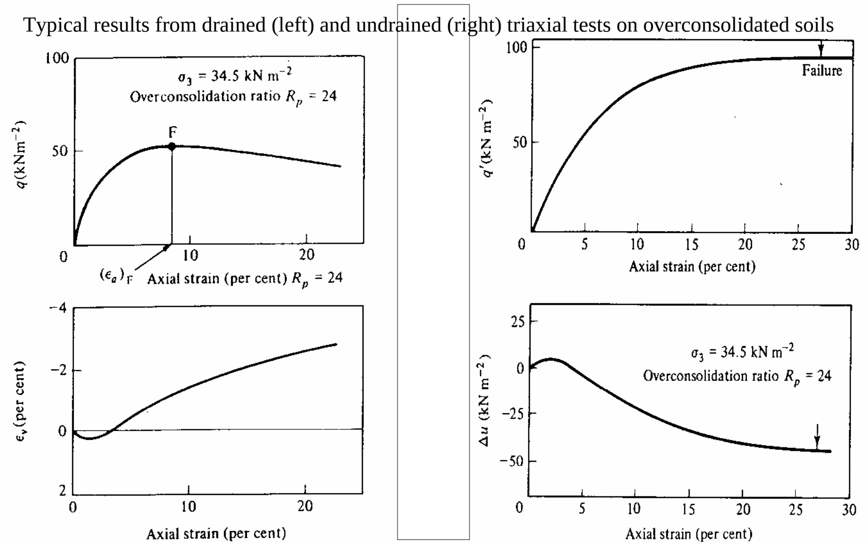
Triaxial test (NC) – drained / undrained

Typical results from drained (left) and undrained (right) triaxial tests on normally consolidated soils (from Atkinson & Bransby, 1978)



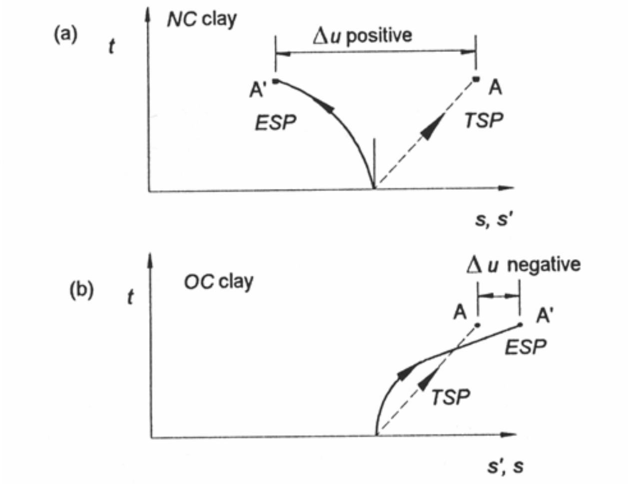
Triaxial test (OC) – drained / undrained

Typical results from drained (left) and undrained (right) triaxial tests on overconsolidated soils



Undrained triaxial test– NC / OC

Typical results from undrained triaxial tests on (a) normally consolidated and (b) overconsolidated clay (from Ortigao, 1995)



Skempton's parameters A and B

Skempton 1954: $$\Delta p_w = B\left[\Delta\sigma_3 + A\left(\Delta\sigma_1 - \Delta\sigma_3\right)\right]$$

- Fully saturated soil
- No inflow / outflow of pore water
- Bulk modulus of soil *grains* is considered to be very high
- Isotropic linear elastic material behaviour (Hooke´s law)

$$\Delta\varepsilon_{vol,\,skeleton} = \Delta\varepsilon_{vol,\,pore\ water}$$

$$\Delta\varepsilon_{vol,\,skeleton} = \frac{\Delta p'}{K'} \qquad K' = \frac{E'}{3(1-2\nu')}$$

$$\Delta\varepsilon_{vol,\,pore\ water} = \frac{n\,\Delta p_w}{K_w}$$

Skempton's parameters A and B

Assuming triaxial compression: $\Delta\sigma_1$; $\Delta\sigma_2 = \Delta\sigma_3$

$$\Delta p_w = \frac{\Delta\sigma_1 + 2\Delta\sigma_3 - 3\Delta p_w}{3K'} \cdot \frac{K_w}{n}$$

leading to $$\Delta p_w = \frac{1}{1+\frac{nK'}{K_w}}\left[\Delta\sigma_3 + \frac{1}{3}(\Delta\sigma_1 - \Delta\sigma_3)\right]$$

$$\Delta p_w = B\left[\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)\right]$$

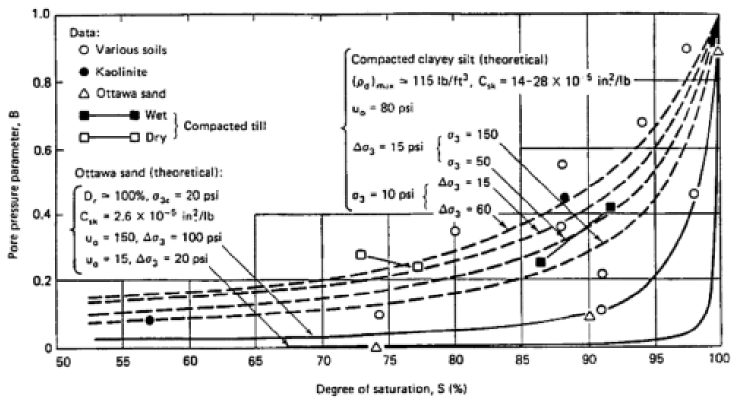
with $$B = \frac{1}{1+\frac{nK'}{K_w}} \qquad A = \frac{1}{3}$$

Skempton's parameters A and B

- Notes on parameters A and B:
  - For K_w large compared to K´, parameter B ~ 1.0 (corresponds to $\Delta p_w = \Delta p > \Delta p' = 0$)
  - Small amount of drapped air reduces parameter B significantly (see next figure)
  - Parameter A depends on stress path, even for elastic material behaviour
  - Parameter A cannot be determined a priori for complex elastic-plastic constitutive models but is a result of the model behaviour for the stress path followed

Skempton’s parameters A and B

Dependence of pore pressure parameter B on degree of saturation



Undrained behaviour with PLAXIS

PLAXIS automatically adds stiffness of water when undrained material type is chosen using the following approximation:

$$\mathbf{K_{total} = K' + \frac{K_w}{n} = \frac{E_u}{3(1-2\nu_u)} = \frac{2G(1+\nu_u)}{3(1-2\nu_u)}}$$

$$\mathbf{K_{total} = \frac{E'(1+\nu_u)}{3(1-2\nu_u)(1+\nu')}}$$ assuming $\nu_u = 0.495$

Notes:

- This procedure gives reasonable B-values only for $\nu' < 0.35$!
- Real value of $K_w/n \sim 1.10^6$ kPa (for n = 0.5)
- In Version 8 B-value can be entered explicitely for undrained materials

Undrained behaviour with PLAXIS

Example 1:

E´ = 3 000 kPa, ν´ = 0.3, ν_u = 0.495
→ K´ = 2 500 kPa, K_{total} = 115 000 kPa → K_w/n = 112 500 kPa

with $$B = \frac{1}{1+\frac{nK'}{K_w}}$$ **= 0.978 > reasonable value for saturated soil**

Example 2:

E´ = 3 000 kPa, ν´ = 0.45, ν_u = 0.495
→ K´ = 10 000 kPa, K_{total} = 103 103 kPa → K_w/n = 93 103 kPa

B = 0.903 > poor value for saturated soil

Undrained behaviour with PLAXIS

Method A (analysis in terms of *effective* stresses):
- type of material behaviour: *undrained*
- **effective** strength parameters c´, φ´, ψ´
- **effective** stiffness parameters E_{50}´, ν´

Method B (analysis in terms of *effective* stresses):
- type of material behaviour: *undrained*
- **undrained** strength parameters c = c_u, φ = 0, ψ = 0
- **effective** stiffness parameters E_{50}´, ν´

Method C (analysis in terms of *total* stresses):
- type of material behaviour: *drained*
- **total** strength parameters c = c_u, φ = 0, ψ = 0
- **undrained** stiffness parameters E_u, ν_u = 0.495

Undrained behaviour with PLAXIS

Notes on different methods:

- Method A:
  - Recommended
  - Soil behaviour is always governed by effective stresses
  - Increase of shear strength during consolidation included
  - Essential for exploiting features of advanced models such as the Hardening Soil model, the Soft Soil model and the Soft Soil Creep model
- Method B:
  - Only when no information on effective strength parameters is avilable
  - Cannot be used with the Soft Soil model and the Soft Soil Creep model
- Method C:
  - NOT recommended
  - No information on excess pore pressure distribution (total stress analysis)

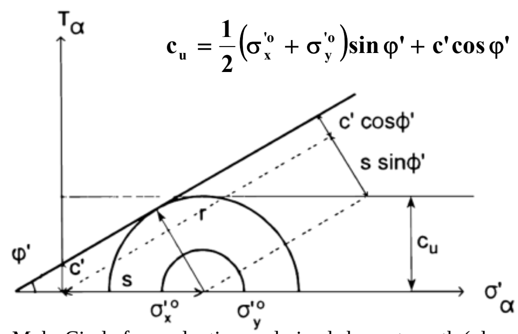
Undrained strength from Mohr circle

Consider fully undrained isotropic elastic behaviour
(Mohr Coulomb in elastic range)

$\Delta p_w = \Delta p \quad > \quad \Delta p' = 0$

$\rightarrow$ centre of Mohr Circle remains at the same point

$$\mathbf{c_u = \frac{1}{2}\left(\sigma'^{o}_{x} + \sigma'^{o}_{y}\right)\sin\varphi' + c'\cos\varphi'}$$



Mohr Circle for evaluating undrained shear strength (plane strain)

Influence of constitutive model

Parameter sets for Hardening Soil model

Model Number	E_{50}^{ref}	E_{ur}^{ref}	E_{oed}^{ref}	■	☼	c	🖫$_{ur}$	p^{ref}	m	K_0^{nc}	R_f
	kN/m²	kN/m²	kN/m²	°	°	kN/m²	-	kN/m²	-	-	-
HS_1	30 000	90 000	30 000	35	0 / 10	0.0	0.2	100	0.75	0.426	0.9
HS_2	50 000	150 000	50 000	35	0	0.0	0.2	100	0.75	0.426	0.9
HS_3	15 000	45 000	15 000	35	0	0.0	0.2	100	0.75	0.426	0.9
HS_4	30 000	90 000	40 000	35	0	0.0	0.2	100	0.75	0.426	0.9
HS_5	30 000	90 000	15 000	35	0	0.0	0.2	100	0.75	0.426	0.9
HS_6	50 000	150 000	30 000	35	0	0.0	0.2	100	0.75	0.426	0.9

Parameters for MC Model

E = 30 000 kN/m²

🖫= 0.2

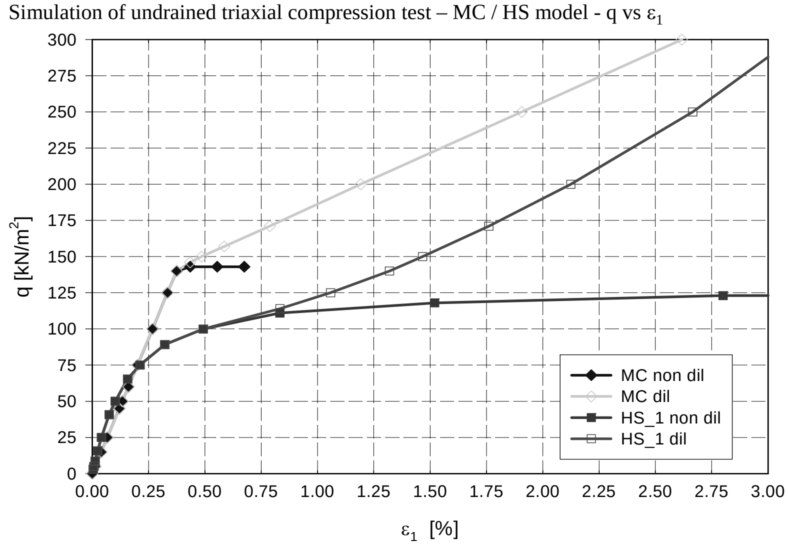
■ = 35°

☼ = 0° and 10°

see also Schweiger (2002)

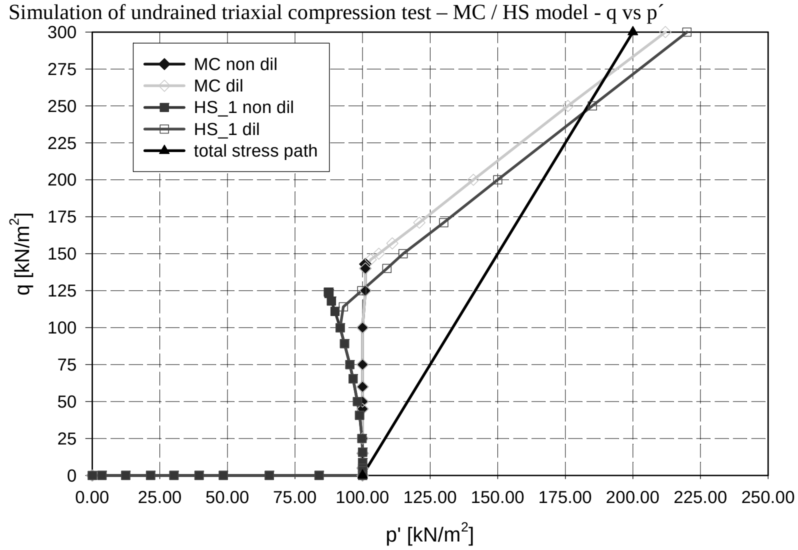
Comparison MC-HS (influence ψ)

Simulation of undrained triaxial compression test – MC / HS model - q vs ε_1



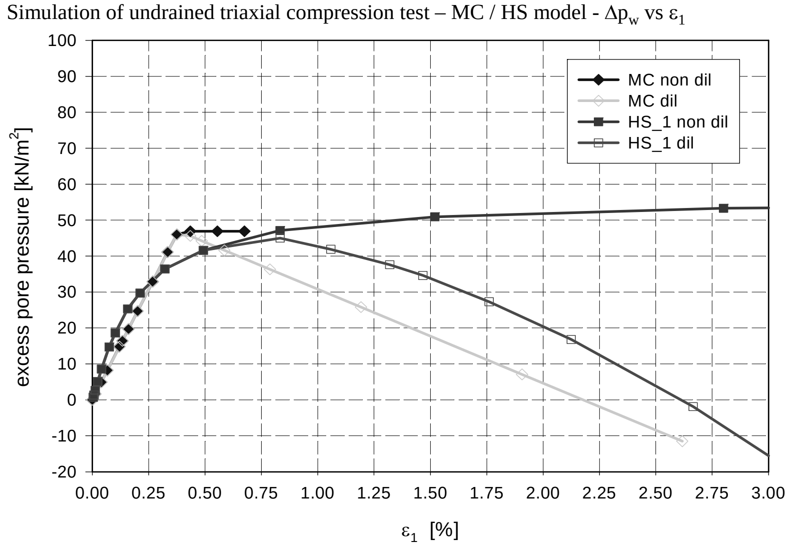
Comparison MC-HS (influence ψ)

Simulation of undrained triaxial compression test – MC / HS model - q vs p´



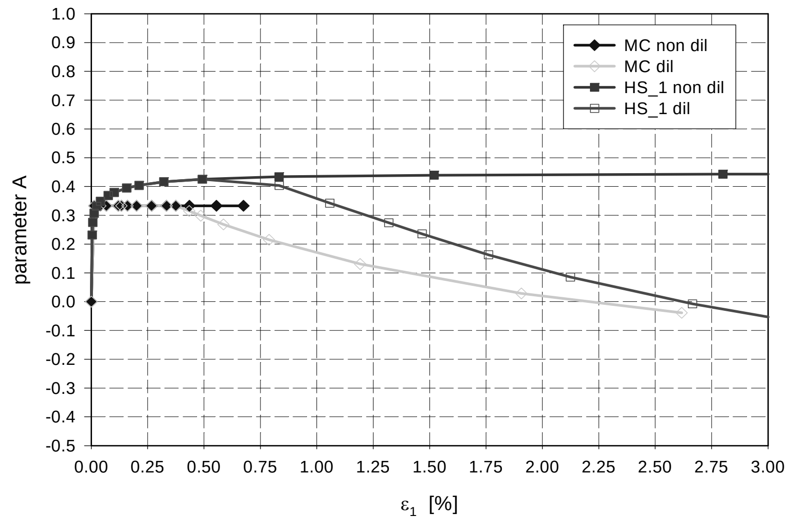
Comparison MC-HS (influence ψ)

Simulation of undrained triaxial compression test – MC / HS model - Δp_w vs ε_1



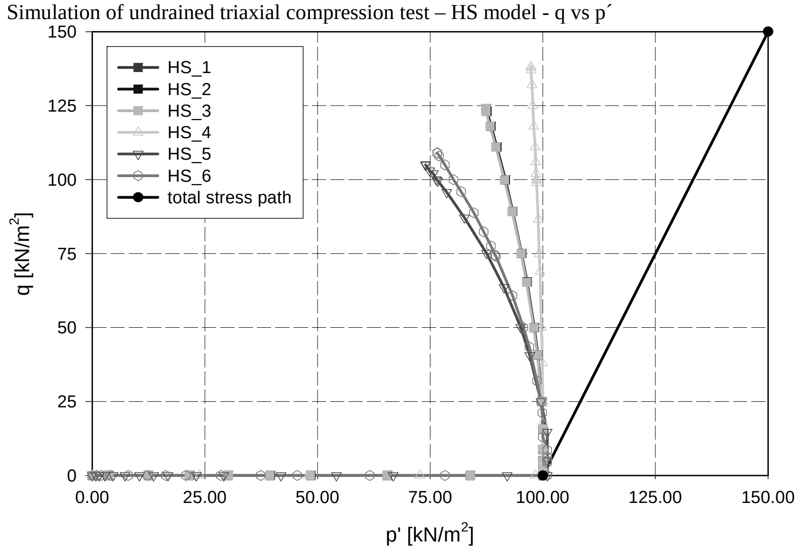
Comparison MC-HS (influence ψ)

Simulation of undrained triaxial compression test – MC / HS model - A vs ε_1



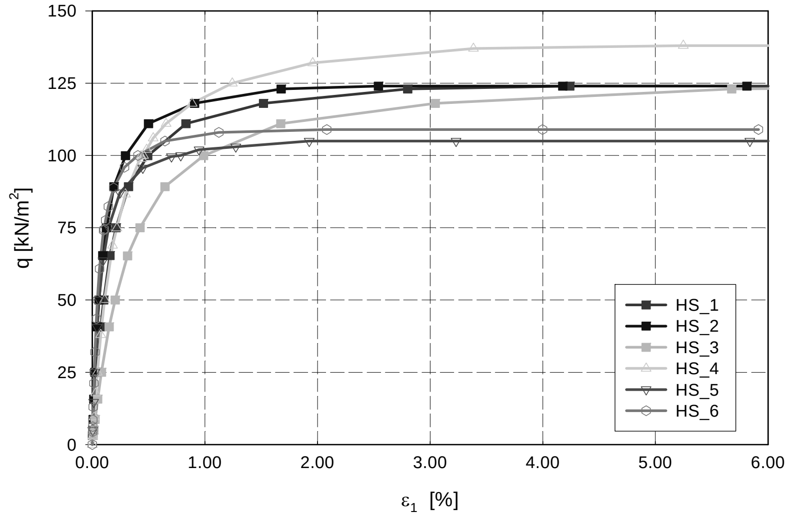
Parameter variation – Hardening Soil

Simulation of undrained triaxial compression test – HS model - q vs p´



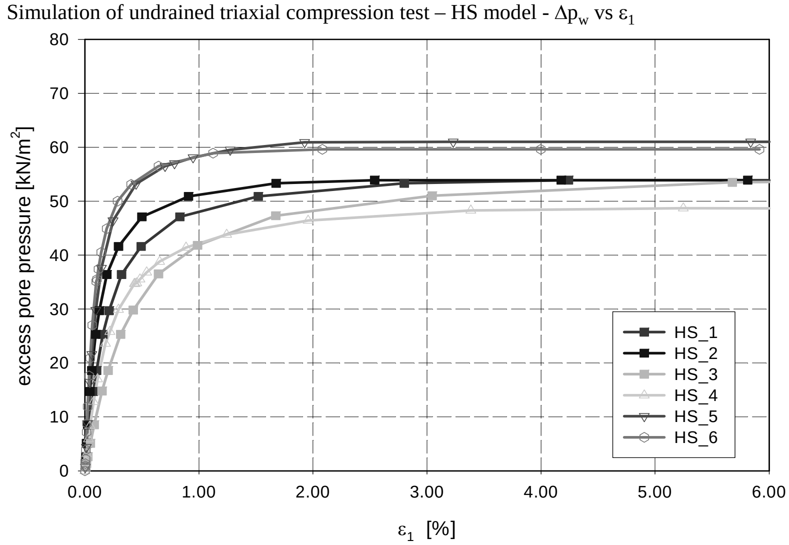
Parameter variation – Hardening Soil

Simulation of undrained triaxial compression test – HS model - q vs ε_1



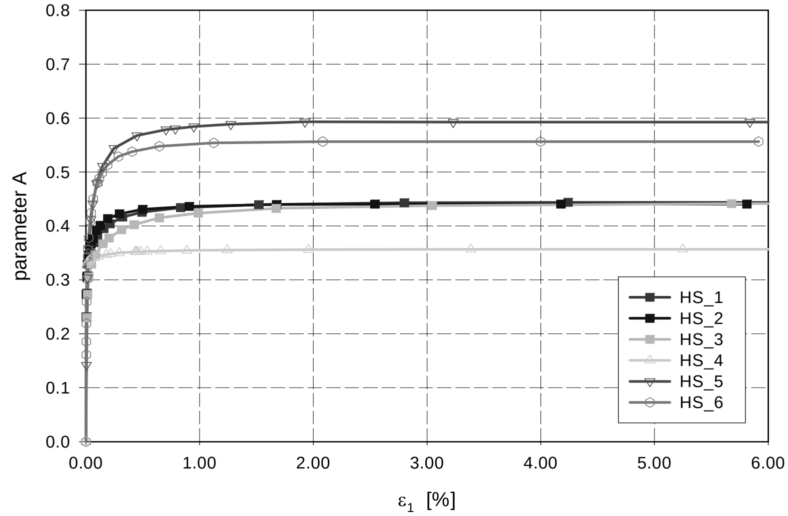
Parameter variation – Hardening Soil

Simulation of undrained triaxial compression test – HS model - Δp_w vs ε_1



Parameter variation – Hardening Soil

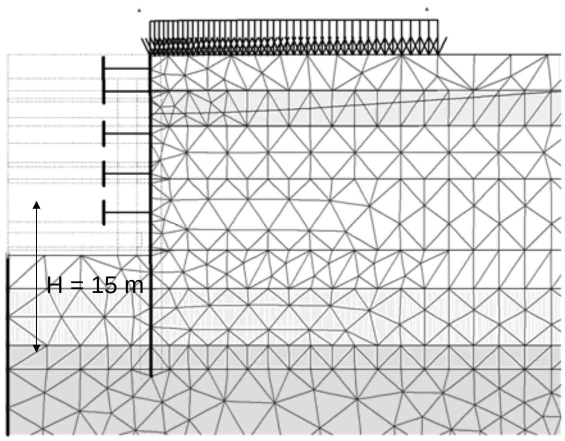
Simulation of undrained triaxial compression test – HS model - A vs ε_1



Summary

- Undrained analysis should be performed in *effective* stresses and with *effective* stiffness and strength parameters
- Undrained shear strength is *result* of the constitutive model
- Care must be taken with choice of value for dilatancy angle
- Note that for NC-soils in general:
  - Factor of safety against failure is *lower for short term* (undrained) conditions for loading problems (e.g. embankment)
  - Factor of safety against failure is *lower for long term* (drained) conditions for unloading problems (e.g. excavations)

Real Excavation Case in Stiff Residual Soils-What is likely Field Conditions?



$$T = \frac{c_v * t}{H^2}$$

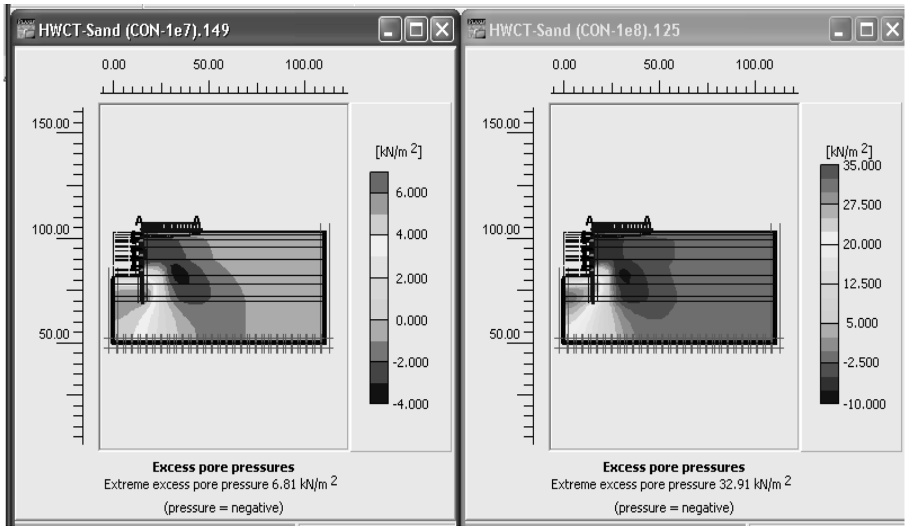
What is West Coast Station Situation ???

Consolidation Considerations in Excavation

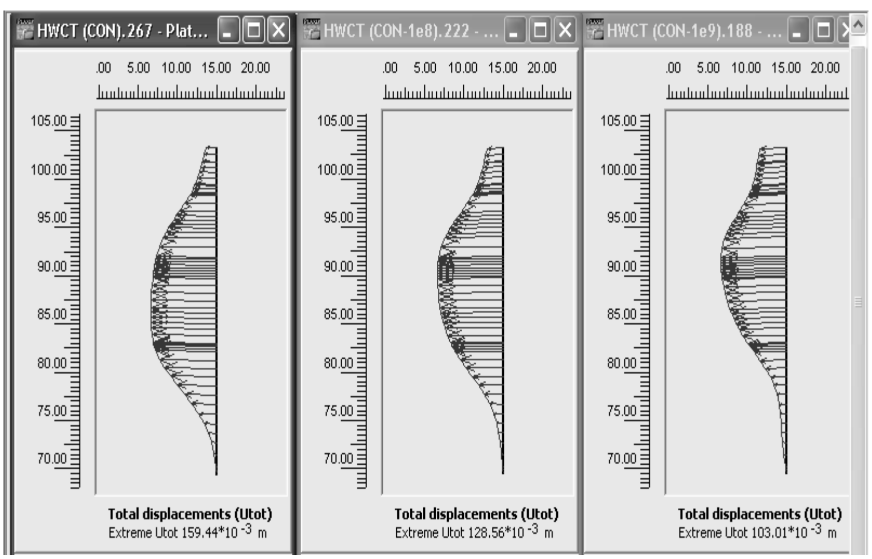
Parameters for West Coast Station

- Cv= 50 m2/day
- H=15 m
- t = 100 days
- T=50*100/(15*15) = 22.2 >>> 0.4
- Situation on Passive Side is likely to be DRAINED Condition

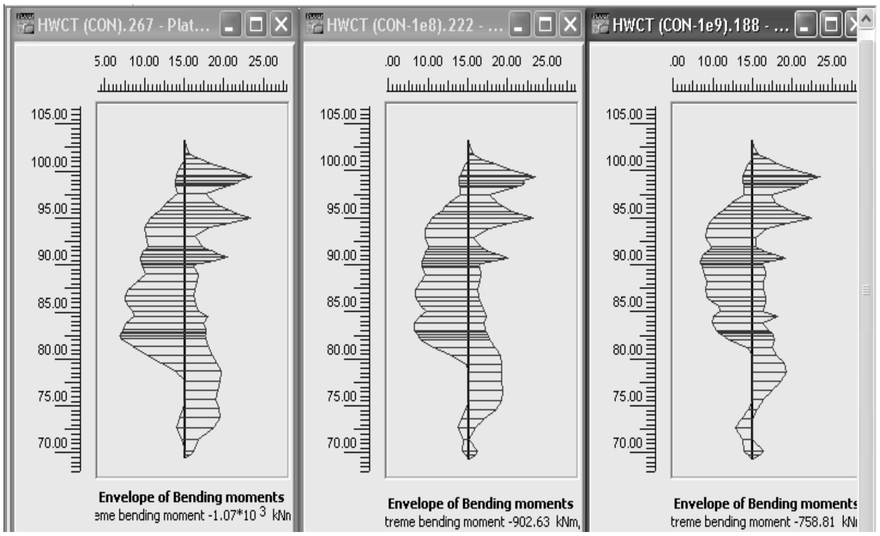
Excess PP at Formation Level for k=1e-7 and 1e-8 m/s



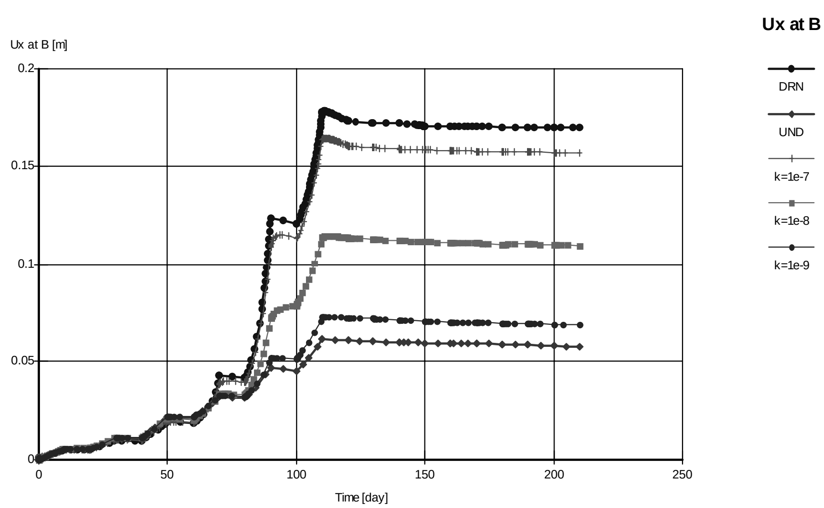
Cases of k=1e-7 to 1e-9 m/s
Displacements at Formation Level



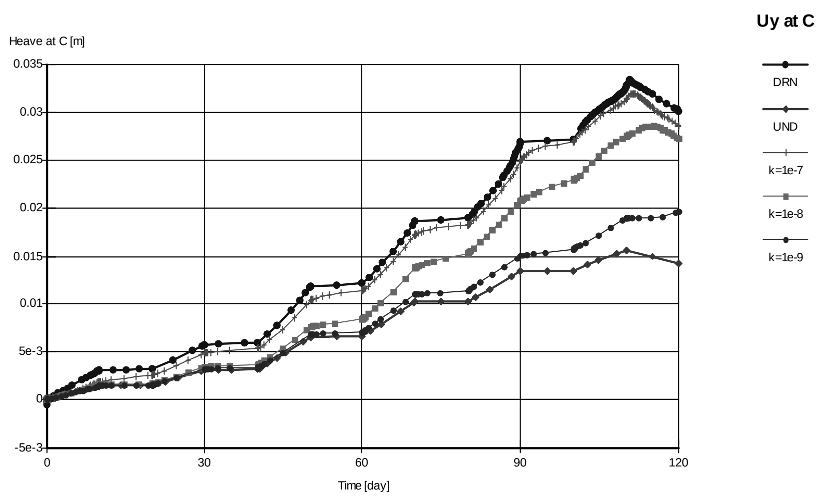
Cases of k=1e-7 to 1e-9 m/s
BMs at Formation Level



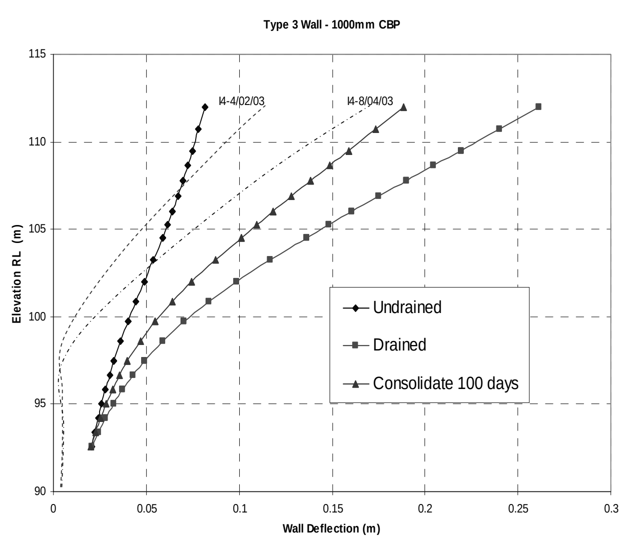
Wall Deflection at B (15/83.85 – 1.65m above FL)



Heave at C(0/78.7 – 3.5m below FL)



Sengkang – Cantilever CBP Wall



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